SQUARE ROOT

SQUARE ROOT of a number is a second number whose product with itself gives the original number. For example, a square root of 4 is 2, because \(2 \times 2 = 4\). The symbol for a square root, called a radical sign, is \(\sqrt{\phantom{0}}\).

For example, \(\sqrt{25} = 5\) and \(\sqrt{4} = 2\). The negative number \(-2\) is also a square root of \(4\), because \(-2 \times -2 = 4\). Each positive number has both a positive and negative square root. These two square roots always have the same numerical value.

Finding Square Roots

By Tables and Slide Rule. An easy method for finding a square root of a number is to use a table of square roots, a table of squares, or a table of logarithms. If available, these tables give a square root quickly and make long and tiresome calculations unnecessary. You can usually learn to use these tables in a short time. An even quicker way of finding a square root consists of using a slide rule. But a slide rule can usually give a square root in only three digits. See LOGARITHMS; SLIDE RULE.

By Dividing and Averaging. If you lack tables, or need greater accuracy than the tables can offer, you must actually compute a square root. You can easily learn, apply, and understand the following method.

Suppose you want to find the square root of 40. Because \(6 \times 6 = 36\) and \(7 \times 7 = 49\), you can see that 6 is closest to \(\sqrt{40}\) in whole numbers. So you begin the square root of 40 with 6. First, divide 40 by 6: \(40 \div 6 = 6.666\overline{6}\) (to the nearest tenth). You can see that \(6 \times 6.666\overline{6} = 39.6\), or about 40. Now find the average of 6 and 6.666\overline{6}: \(\frac{1}{2}(6+6.666\overline{6}) = 6.333\overline{3}\). This is even closer to 40.

To obtain greater accuracy, repeat the procedure. First, divide 40 by 6.333\overline{3}: \(40 \div 6.333\overline{3} = 6.349\). Next, find the average of 6.3 and 6.349: \(\frac{1}{2}(6.3+6.349) = 6.3245533\). Repeating the procedure a third time, you find that \(40 \div 6.3245533 = 6.3245533\), and that \(\frac{1}{2}(6.3245533+6.3245533) = 6.3245533\).

You may continue this process as long as you wish. In general, in each approximation to the square root you usually keep twice as many digits as in the previous approximation.

Notice that 40 lies between 1 and 100. If you want to find the square root of a number that is not in the 1 to 100 range, you must first divide or multiply it by 100 to bring it within this range. Suppose you want to find the square root of 400,000, or \(\sqrt{400,000}\). You must divide 400,000 twice by 100. This gives you 40, a number within the 1 to 100 range. You know how to find the square root of 40: \(\sqrt{40} = 6.3245533\). Now you must multiply the square root of 40 twice by 10 (the square root of 100) to obtain the square root of 400,000: \(6.3245533 \times 10 \times 10 = 632.45533\). In the same way, \(\sqrt{0.4} = 0.63245533\). You find the square root of 0.4 by multiplying by 100, finding the square root of 40, and dividing by 10.

A Method Often Used in School

Besides the above, you can use other processes for finding square roots. Here is a method often taught in schools.

\[
\text{Find } \sqrt{1248.4}
\]

Step 1. Using little marks, separate 1,248.4 into two-digit periods, or units, in each direction from the decimal point. One digit of the square root will correspond to each two-digit period.

\[
\sqrt{1248.40}
\]

Now determine the largest square, or number multiplied once by itself, in 12, the leading two-digit period. The largest square in 12 is 3\(\times\)3, or 9. Write 3 as the first digit of the square root, subtract 9 from 12, and bring down the next period to form the remainder 348:

\[
\begin{array}{c|c}
\text{Step 1} & 3 \\
& \sqrt{1248.40} \\
& 9 \\
& 3 \frac{48}{348} \\
\end{array}
\]

Step 2. Multiply the partial square root, 3, by 2, and place the product, 6, to the left of 348. Determine the next digit of the square root by dividing 348 by 10\(\times\)6, or 60: \(348 \div 60 = 5\). Write 5 as the next digit of the square root and annex 5 to the 6 to form the divisor 65. No multiply 65 by 5, subtract the product from 348, and bring down the next period to form the remainder 2,340:

\[
\begin{array}{c|c}
\text{Step 2} & 3 \frac{5}{3} \\
& \sqrt{1248.40} \\
& 9 \\
& 65 \frac{348}{325} \\
& 325 \frac{2340}{2310} \\
\end{array}
\]

Step 3. Multiply the partial square root, 35, by 2, and place the product, 70, to the left of 2,340. Determine the third digit of the square root by dividing 2,340 by 10\(\times\)70, or 700: \(2340 \div 700 = 3\). Write 3 as the third digit of the square root and annex 3 to the 70 to form the divisor 703. Multiply 703 by 3, subtract the product from 2,340, and bring down the next period to form the remainder 23,100:

\[
\begin{array}{c|c}
\text{Step 3} & 3 \frac{5.3}{35} \\
& \sqrt{1248.40} \\
& 9 \\
& 65 \frac{348}{325} \\
& 703 \frac{2340}{2109} \\
& 2109 \frac{23100}{23100} \\
\end{array}
\]

You may continue this process as long as you wish. Place the decimal point in the square root over the decimal point in the given number.

Square Roots of Negative Numbers

What is the square root of \(-4\)? Or, what number multiplied by itself gives a product of \(-4\)? If there is such a number, it certainly cannot be positive, negative, or zero. None of these multiplied by itself can give a negative number. But, for convenience in solving certain problems, mathematicians have invented a system of pure imaginary numbers whose squares are negative numbers.

See also CUBE ROOT; ROOT; SQUARE.