

# Finding roots to second-degree polynomial equations

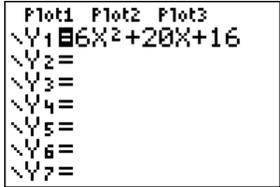
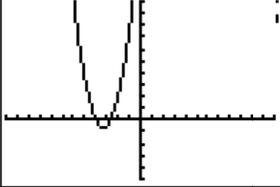
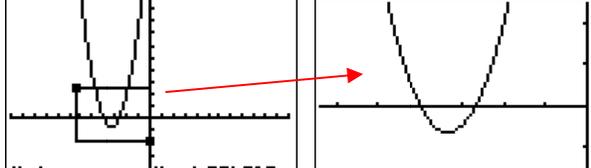
*Three methods: By Groups, Graphing Calculator, and Quadratic Formula*

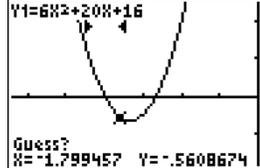
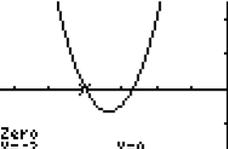
Given:  $6x^2 + 20x + 16 = 0$

Method 1: , By Groups. Useful if you don't have a calculator, but time consuming, many steps.

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| 1 | $6 \times 16 = 96$   | Multiply the first and last coefficients, 6 and 16 together   |
| 2 | 12 & 8. $12 \times 8 = 96$ , and $12 + 8 = 20$                           | What numbers multiply to 96 and add to 20?  |
| 3 | $6x^2 + 12x + 8x + 16$   | Re-write equation using 12x & 8x instead of 20x   |
| 4 | $(6x^2 + 8x) + (12x + 16)$ (remember, 8x & 12x is 20x split up)          | Group. Note, $(6x^2 + 12x) + (8x + 16)$ is another way to group, but causes a problem when checking. At this step, grouping can only be done two ways. This is the only step of "guess and check" |
| 5 | $2x(3x + 4) + 4(3x + 4)$   | Factor out- make sure the "(3x + 4)" are the same   |
| 6 | $(2x + 4)(3x + 4)$   | Re-write, combine terms   |
| 7 | Does $(2x + 4)(3x + 4) = 6x^2 + 20x + 16$ (?)                            | Check- yes it does.   |
| 8 | Calculate the roots. $(2(-2) + 4) = 0$ , and $(3(\frac{-4}{3}) + 4) = 0$ | Roots are -2, and -4/3  |

Method 2: TI-83/ TI-84 graphing calculator. Allows you to see the equation visually, but requires proficiency with a TI calculator. Answers are displayed as a decimal, not a fraction.

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|---|--|
|  | <p>Type in the equation in the Y= screen.</p>  |
|  | <p>View the graph. The function crosses the x-axis at two places, which means two places where <math>y = 0</math>. These are the roots. Use the Zoom Box to get a closer look.</p> |
|  | <p>Using the "zoom" key, choose "ZBox" which is menu item 1. Form a box around the lower portion of the curve. The results should look similar to the picture on the right.</p>    |

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|---|--|--|---|
|   | <pre> 2: CALC 1: value 2: zero 3: minimum 4: maximum 5: intersect 6: dy/dx 7: ∫f(x)dx </pre>   |  | <p>To find the roots, use the “calc” menu above the TRACE key. Choose item 2, “zero.”</p>   |
|  <p>Y1=6X<sup>2</sup>+20X+16<br/>Left Bound?<br/>X=-2.172929 Y=.87114219</p> |  <p>Y1=6X<sup>2</sup>+20X+16<br/>Guess?<br/>X=-1.799457 Y=-.5608674</p> |  | <p>The calculator needs to know a boundary for which zero to calculate, because there are two. We'll find the left-most root first. Select a left and right on either side of the left-most root.</p> |
|   |  <p>Zero<br/>X=-2 Y=0</p>   |  | <p>One of the two roots is found to be -2. Repeat the same procedure for the right-most root. The result will be displayed as -1.333... which is -4/3</p>   |

Method 3: Quadratic formula. Quick & easy, but may require a calculator with a square-root key. Mistakes are often made because of the complex order-of-operations in the formula.

|   |   |
|---|---|
| $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$                  | <p>The formula for solving a quadratic equation.</p>  |
| $x = \frac{-20 \pm \sqrt{20^2 - 4(6)(16)}}{2(6)}$         | <p>Substitute numbers for the letters. a = 6, b = 20, and c = 16. Compute both positive and negative roots, note the symbol “±” means there is a positive and negative operation (two in one)</p> |
| $x_1 = \frac{-20 + 4}{12} \quad x_2 = \frac{-20 - 4}{12}$ | <p>Clean up all the exponents and radicals, and your two roots are within reach...</p>  |
| $x_1 = -\frac{4}{3} \quad x_2 = -2$                       | <p>Here are the two roots</p>   |